

Screen, Project & Fold; Planning a DoE Project

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Planning of this talk on planning

- Introduction
 - DoE strategy
 - Confounding
 - Projection and fold-over
- Pathways for projection and fold-over
 - Regular Fractional Factorials
 - Minimum Run
- Conclusions



Strategy for DoE

- Goal - Model - Design
 - Range check
 - **Screening / Exploration**
 - **model: main effects**
 - **Design: 2-level**
 - **Quantification**
 - **model: 2FI**
 - **Design: 2-level**
 - RSM (Optimization)
 - ...
- Sequential experimentation



2-level Design Options

- *Full Factorial*
 - *Rich, but too large for >4 factors*
- **Fractional Factorials**
 - *May still be too large*
- **Minimum Run**
 - *nice and small*
 - *not fully orthogonal*
 - *Some increase in VIF / Condition number / Correlations*
- *Plackett-Burman*
 - *superceded by MR*
- *Optimal Design*
 - *MR are optimal, but in general: hard to control confounding*
- ...



Confounding and Resolution

- Confounding (aliasing)

- Example: 2^{4-1}

[A] = A + BCD
[B] = B + ACD
[C] = C + ABD
[D] = D + ABC
[AB] = AB + CD
[AC] = AC + BD
[AD] = AD + BC

- Resolution

- III

- Main effects confounded with 2FI
- Range check design, interpretation a gamble

- IV

- Main effects free from 2FI

- **Screening**

- V+

- 2FI free from each other

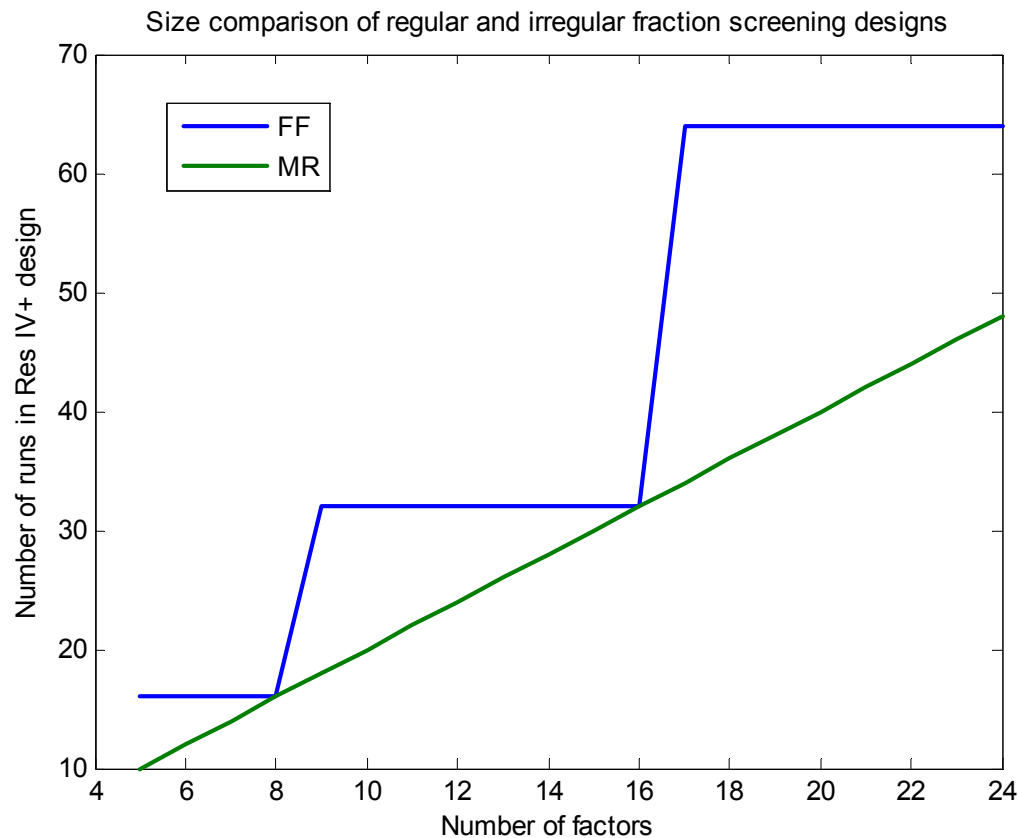
- **Quantification**

- large, even with MR



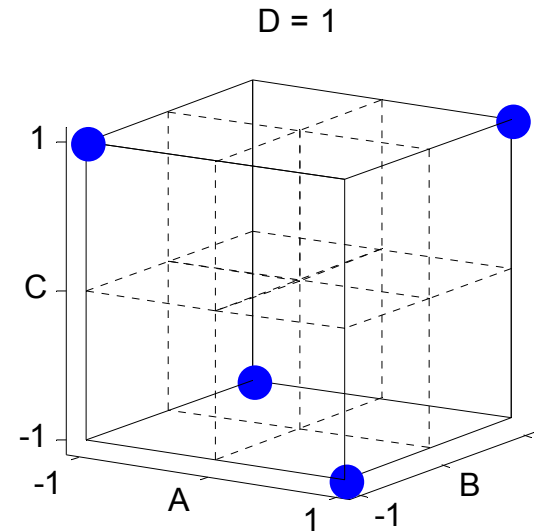
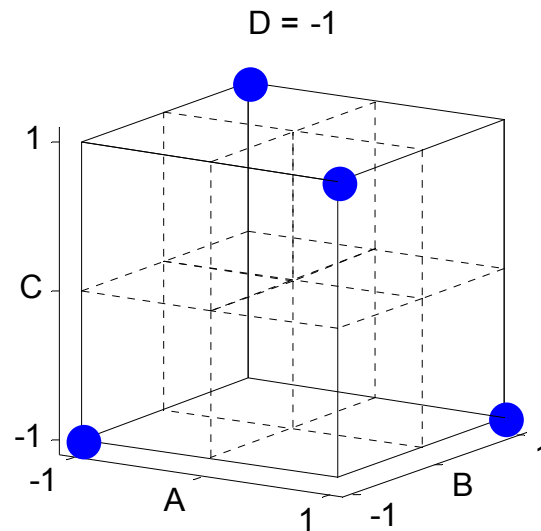
Comparison of FF and MR IV size

- MR IV gives useful reduction in numbers of runs compared to regular fractional factorials



Projection; quantify with screening data

- Assuming
 - Screening works (Pareto !)
 - at least for a single response ...
 - Heridity principle
 - if a factor's main effect is negligible, so are its higher orders
- If few factors remain, design may be full factorial for those
 - example: 2^{4-1}

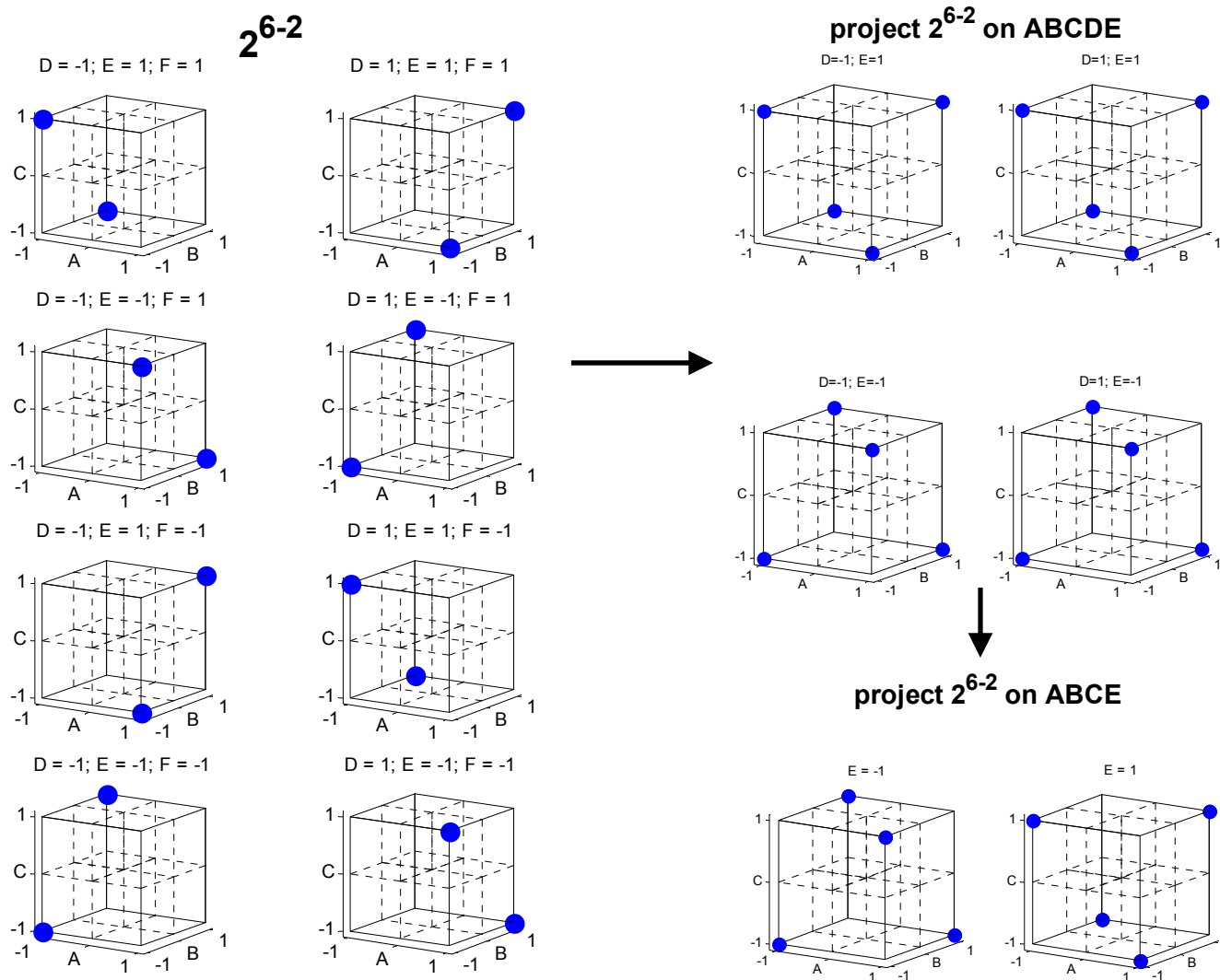


Projection; some rules of magic

- 2^{k-p} designs of Res **R** project into
 - **full factorials** for **(R-1)** factors
 - **full factorials** in **R** factors not appearing as a word in a defining relation
 - **fractional factorials** in **R** factors otherwise
- Example:
 - 2^{7-3} Res IV design
 - Design generators
 - $E = ABC$, $F = BCD$ and $G = ACD$
 - Defining relations
 - $I = ABCE = ABFG = ACDG = ADEF = BCDF = BDEG = CEFG$
 - **Res IV** in any of these combinations of **4** factors
 - Full factorial in
 - the **28 other combinations of 4** factors
 - **all combinations of 3** factors



Projection if the magic fails



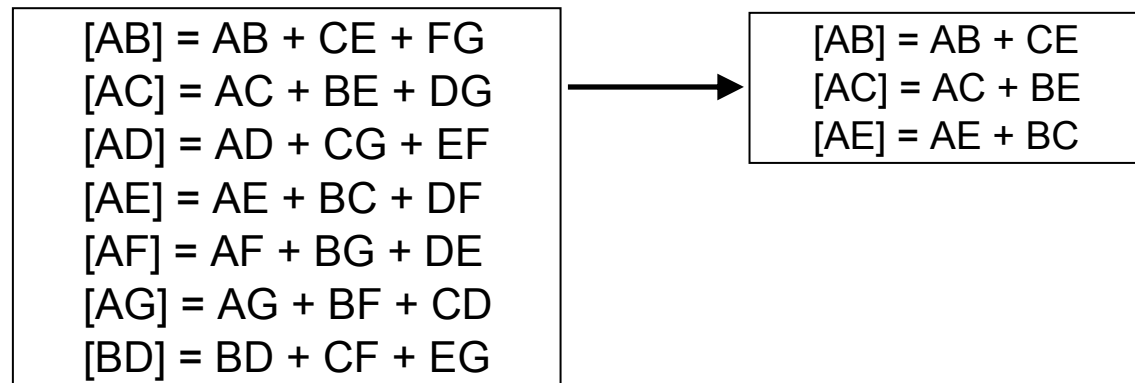
Still Res IV after projection onto 4 factors

N	Design	Defining words of length 4	P _{bad luck}
16	2^{6-2}	ABCE, ADEF, BCDF	3 / 15
	2^{7-3}	ABCE, ABFG, ACDG, ADEF, BCDF, BDEG, CEFG	7 / 35
	2^{8-4}	ABCE, ABDH, ABFG, ACDG, ACFH, ADEF, AEGH, BCDF, BCGH, BDEG, BEFH, CDEH, CEFG, DFGH	14 / 70
32	2^{7-2}	DEFG	1 / 21
	2^{8-3}	ABCF, ABDG, CDFG	3 / 56
	2^{9-4}	BCHJ, BDGJ, BEFJ, CDGH, CEFH, DEFG	6 / 126
	2^{10-5}	ABJK, ACHK, ADGK, AEFK, BCHJ, BDGJ, BEFJ, CDGH, CEFH, DEFG	10 / 252
	2^{11-6}	ABCF, ABGJ, ABKL, ACDJ, ACHK, ADEK, ADFG, AEHJ, AFHL, BCDG, BCHL, BDEL, BDFJ, BEGH, BFHK, CDEH, CEGJ, CEJK, CFGJ, CFKL, DGHL, DHJK, EFGK, EFJL, GJKL	25 / 462



What if still resolution IV for 4 remaining?

- Confounding **will** be less severe
 - example 2^{7-3} where ABCE remain
 - Aliasing before and after projection



- Note we'll ignore anything beyond 2FI
- **Fold-over** can undo some or all remaining confounding of 2FI
- *Clean up using subject matter knowledge*
 - *Some 2FI may be ruled out as unlikely*
 - *Rarely possible*



Fold-over saves the day

- Fold = change sign on a factor (once scaled to [-1 +1])
- Additional experiments
- Complete fold-over
 - Makes disappear odd-length defining relations
 - Res IV remains Res IV, but power increases
 - Only useful to move from resolution III to IV
- Partial fold-over: fold on single factor
 - Changes sign on even-length defining relations involving that factor
 - Undoes (part of) the Res IV confounding
- Semifold: half of a partial
 - Nice and small
 - Not fully orthogonal
 - Some increased in VIF / SE / Condition Number / Correlations



Partial and semi fold, example

- After a 2^{7-3} (**16 runs**), the factors ABCE remain.
 - Hence the defining relation ABCE remains
 - Remaining confounding
- Fold-over options
 - Partial Fold-over (**16 runs**) on, say, A
 - Will make the defining relation ABCE disappear
 - 2FI (and 3FI) free from each other
 - *only the interaction ABCD will be lost to a block*
 - Semifold (**8 runs**) on, say, A, and fold on, say, D+
 - 2FI free from each other
 - But correlations (of size 0.5)
 - VIF = 1.5
 - SE coefficients = 0.25 (vs 1.7 for partial)
 - Somewhat lower power

$$[AB] = AB + CE$$

$$[AC] = AC + BE$$

$$[AE] = AE + BC$$



Planning for regular fractions

- Project & fold pretty straightforward for regular fractions
 - Estimating 2FI for 5 factors always possible

N factor	Fractional IV		+Semi	MR V	Generators	N 4FI	n	Res	N 4FI def rel		
	Code	N							Project	Semi	
6	2 ⁶⁻²	16	24	22	E = ABC F = BCD	15	3	Full			
							4	Full			
								IV	1	0	
								5	IV	1	0
								6	IV	3	1
7	2 ⁷⁻³	16	24	30	E = ABC F = BCD G = ACD	35	3	Full			
							4	Full			
								IV	1	0	
								5	IV	1	0
								6	IV	3	1
						7	IV	7	3		
8	2 ⁸⁻⁴	16	24	38	E = ABC F = BCD G = ACD H = ABD	70	3	Full			
							4	Full			
								IV	1		
								5	IV	1	0
								6	IV	3	1
						7	IV	7	3		
						8	IV	14			



Planning for MR IV screening designs

- Project & fold less obvious for MR, because
 - No clear-cut defining relations
 - Always some correlation
 - Requires other measures of confounding
 - VIF, SE of coefficients, condition number, correlation matrix ...
 - Way too many combinations to do by hand in Design Expert

		<i>Number of factors in the screening design</i>										
		6	7	8	9	10	11	12	13	14	15	16
<i>Number of factors remaining after screening</i>	4	15	35	70	126	210	330	495	715	1001	1365	1820
	5	6	21	56	126	252	462	792	1287	2002	3003	4368
	6		7	28	84	210	462	924	1716	3003	5005	8008
	7			8	36	120	330	792	1716	3432	6435	11440
	8				9	45	165	495	1287	3003	6435	12870

- We used Matlab to systematically assess projectivity for the designs made using Design Expert
 - also for regular fractions with semifold



Criteria of confounding / correlation

- We used the following criteria
 - Rank loss for full 2FI model
 - The number of terms that cannot be estimated
 - E.g. the 2^{4-1} design has a rank loss of 3
 - Rank loss > 0 : no estimation of full 2FI
 - Rank loss = 0 : estimation possible, but maybe bad
 - Max SE of coefficients
 - Max because the SE may be different
 - Calculate if rank loss = 0
 - Gives same picture as max VIF
 - Condition Number of $X'X$
 - Calculate if rank loss = 0
 - Indicates potential for interpretation problems
 - Hard to interpret
- Interesting (but we didn't calculate yet)
 - Min power
 - Linked to SE / VIF

[A]	=	A + BCD
[B]	=	B + ACD
[C]	=	C + ABD
[D]	=	D + ABC
[AB]	=	AB + CD
[AC]	=	AC + BD
[AD]	=	AD + BC



Going over Possible Projections

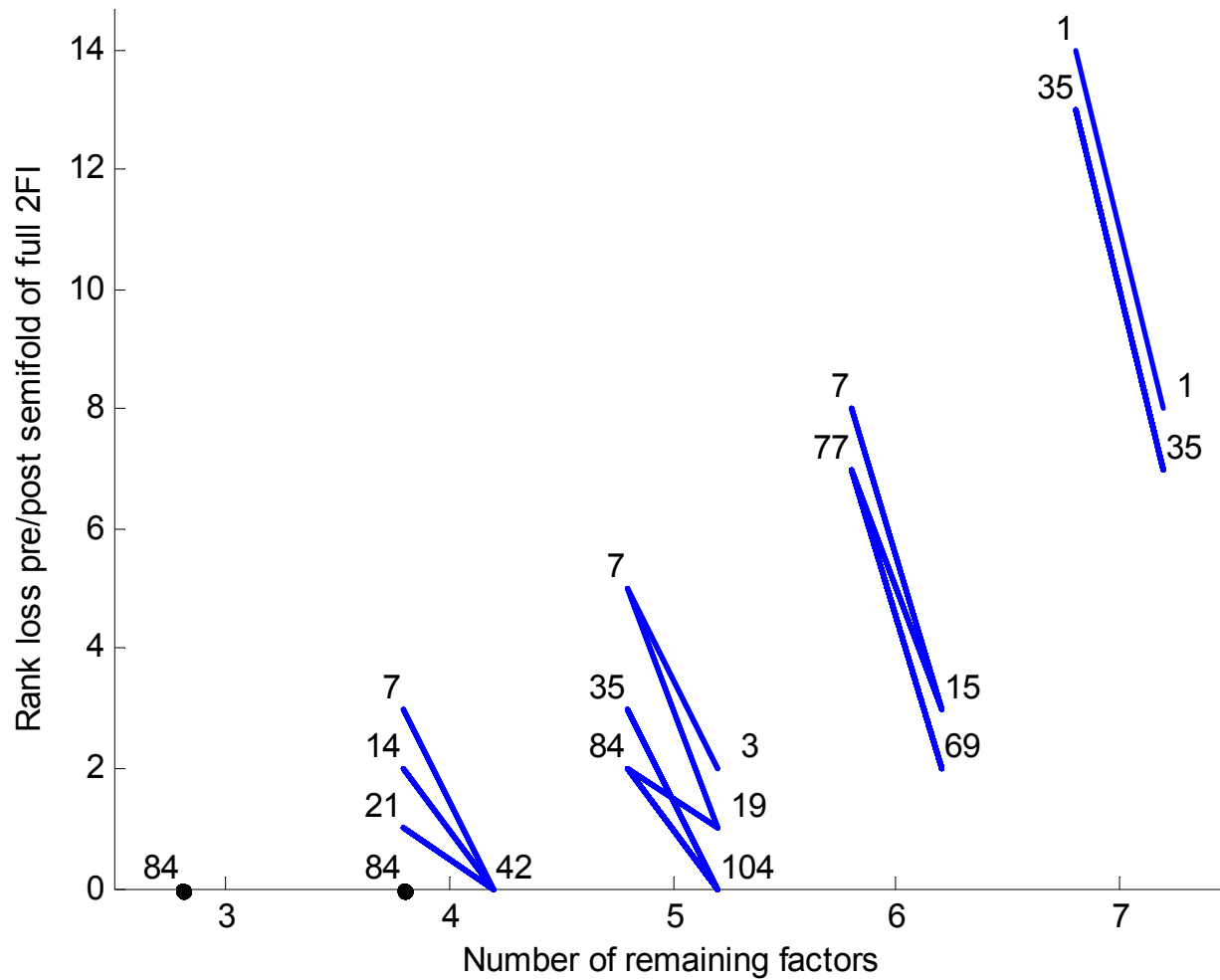
- For all possible subsets of 3, 4, 5 ... factors
 - Project and calculate Rank Loss
 - If Rank Loss = 0, calculate Max SE and Condition Number
 - In case Rank Loss >0 (i.e. still res IV)
 - Check which 2FI are “not estimable” due to confounding
 - E.g. CD, BD and BC in a situation like a 2^{4-1}
- Semi fold
 - Fold on most frequently occurring factor in these 2FI
 - Cut on least frequently occurring
- Calculate Rank Loss
 - If Rank Loss = 0, calculate Max SE and Condition Number

[A] = A + BCD
[B] = B + ACD
[C] = C + ABD
[D] = D + ABC
[AB] = AB + CD
[AC] = AC + BD
[AD] = AD + BC

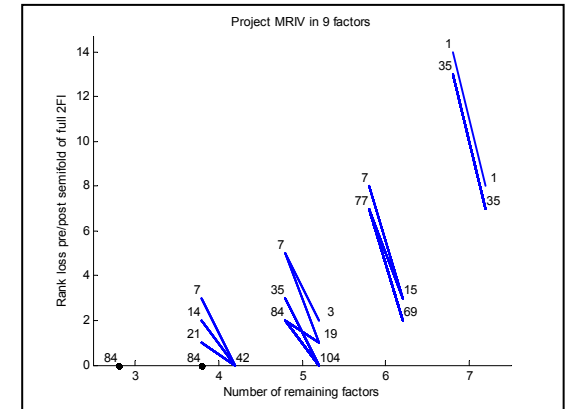
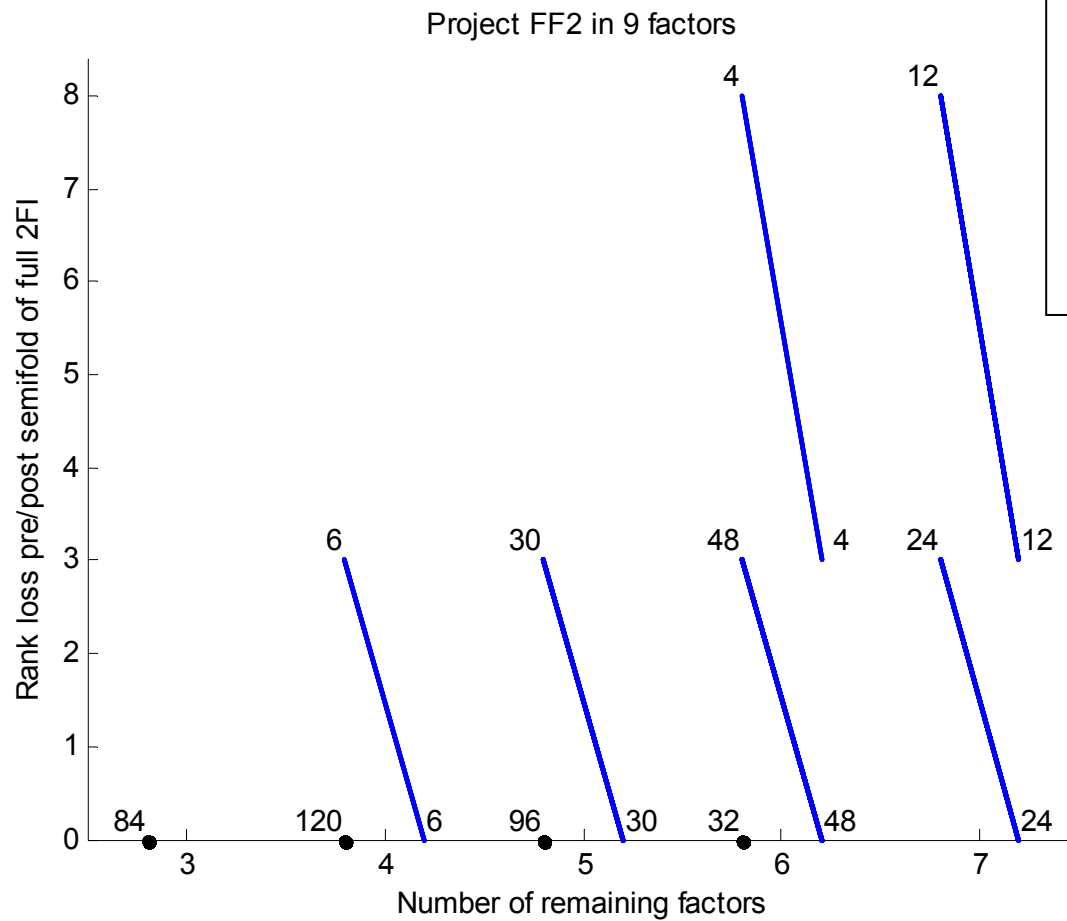


Project&Fold example; Rank Loss

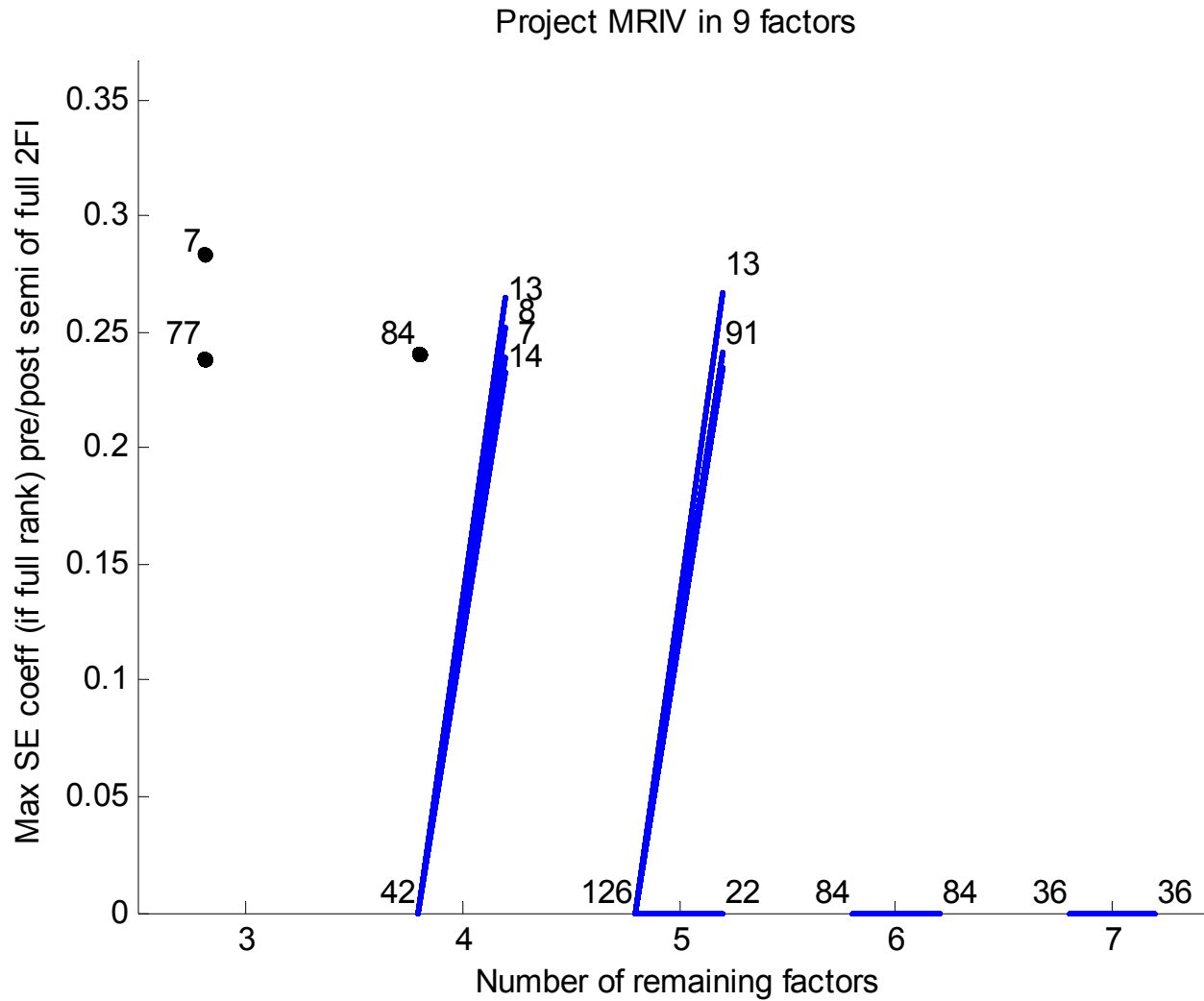
Project MRIV in 9 factors



Rank Loss for regular fraction

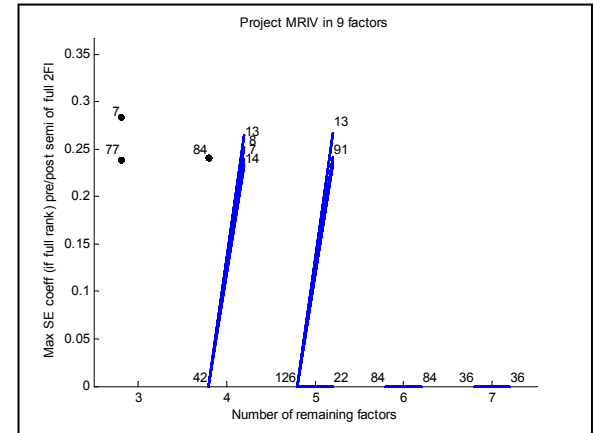
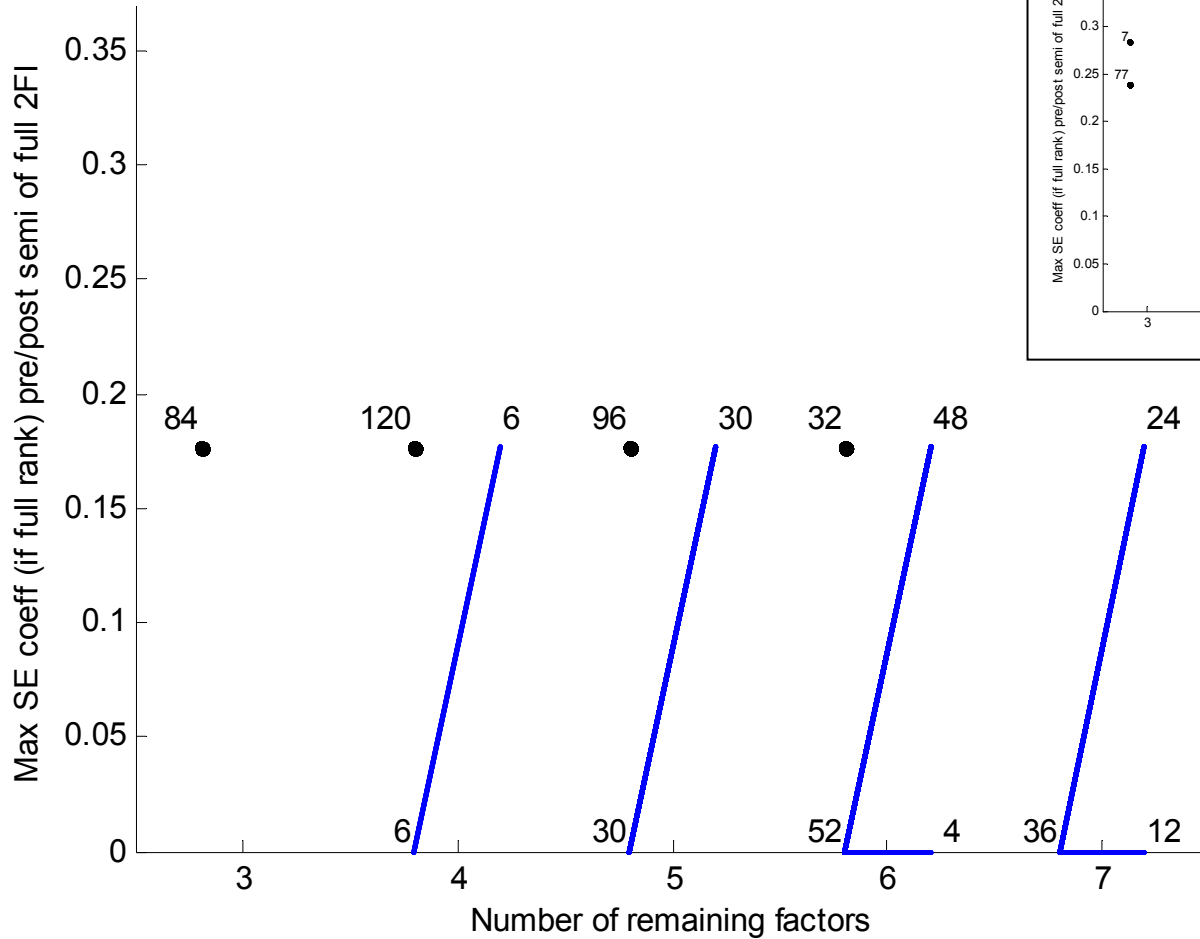


Max SE for MR



Max SE for regular fraction

Project FF2 in 9 factors



Conclusions

- MR IV projects well
 - though not as well as a regular fraction
 - projection on 3 always works without fold
 - projection on 5 nearly always works if folded
 - projection on 8 doesn't, even after (semi) fold
 - checked for designs of upto 16 factors
- Plan for total number of experiments
 - Consider FF or MRIV screening plus project and possibly semifold, or MRV straight away (generally too large)
- Remember the total number of runs will be plus 4
 - repeats in the center are very important
- Extending the design with a (semi)fold may require blocking
- Nice to have: project&fold paths offered in the software ?...

