



# Design and Analysis of Industrial Strip-Plot Designs

Peter Goos

Universiteit Antwerpen





# Outline

- examples
  - battery cell experiment
  - cheese making experiment
  - polypropylene experiment
  - mixture process-variable experiments
- origin
- differences with
  - split-plot designs
  - split-split-plot designs
- model and estimation
- designs



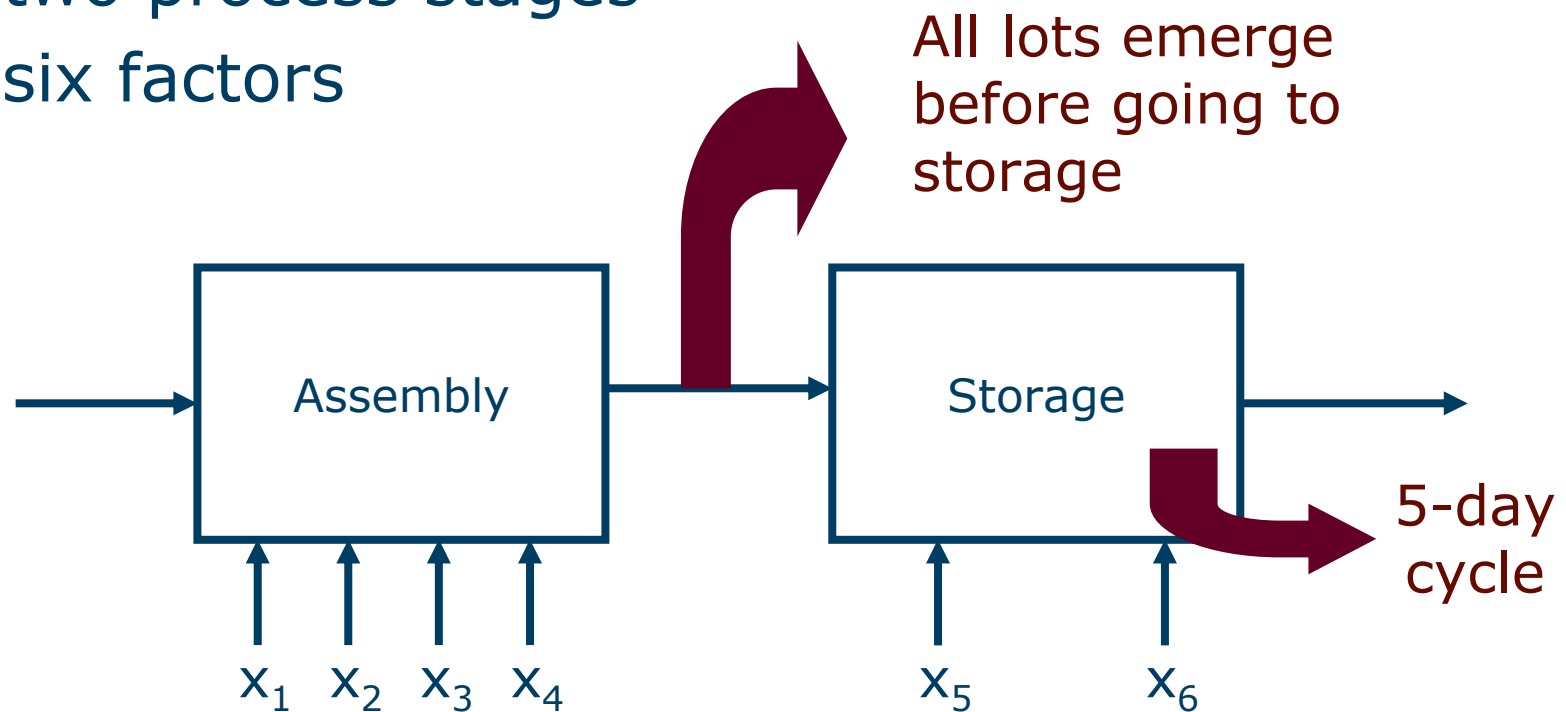
# Terminology

- strip-plot designs
- split-block designs
- strip-block designs
- two-way whole-plot designs
- criss-cross designs
- ...





















# Battery-cell experiment

- two process stages
- six factors





# Battery-cell experiment

Run	Assembly Variables ( $2^4$ )	Storage Variables ( $2^2$ ) Storage Conditions			
		(1)	(2)	(3)	(4)
(1)	 → 				
(2)	 → 				
⋮	⋮	⋮	⋮	⋮	⋮
(16)	 → 				



Assembly Factors				Storage Factors			
				-	+	-	+
X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	-	-	+	+
-	-	-	-	X	X	X	X
+	-	-	-	X	X	X	X
-	+	-	-	X	X	X	X
+	+	-	-	X	X	X	X
-	-	+	-	X	X	X	X
+	-	+	-	X	X	X	X
-	+	+	-	X	X	X	X
+	+	+	-	X	X	X	X
-	-	-	+	X	X	X	X
+	-	-	+	X	X	X	X
-	+	-	+	X	X	X	X
+	+	-	+	X	X	X	X
-	-	+	+	X	X	X	X
+	-	+	+	X	X	X	X
-	+	+	+	X	X	X	X
+	+	+	+	X	X	X	X

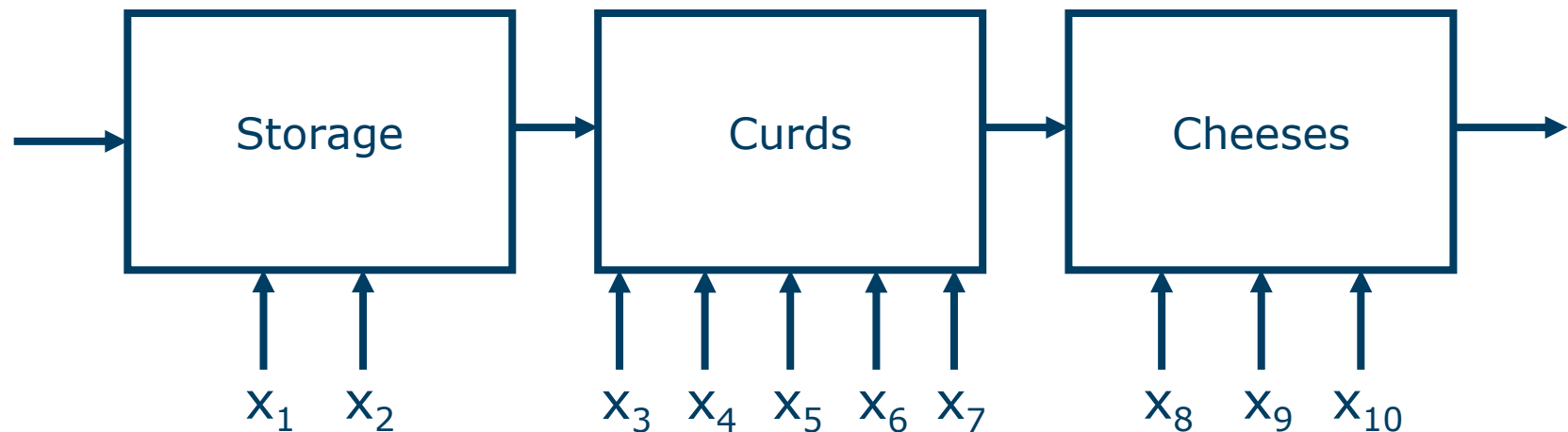
X<sub>5</sub>

X<sub>6</sub>



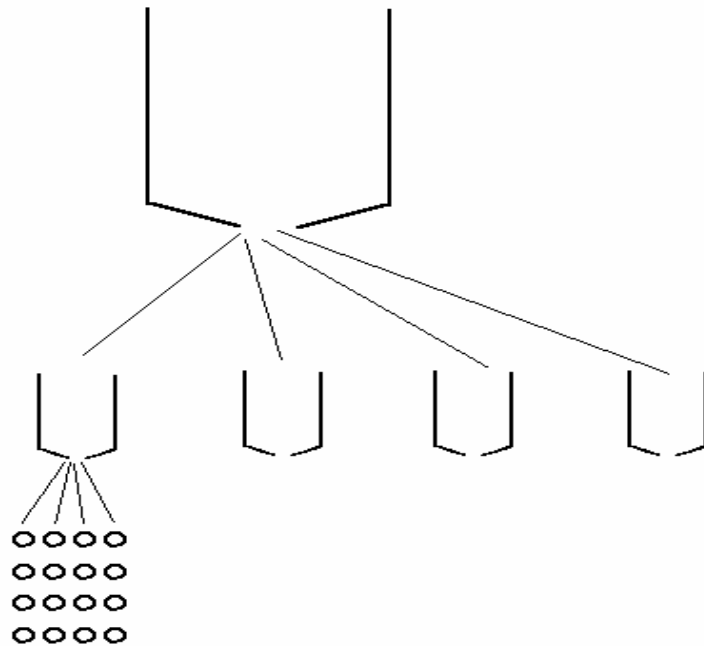
# Cheese-making experiment

- three process stages
- ten factors





# Cheese-making experiment



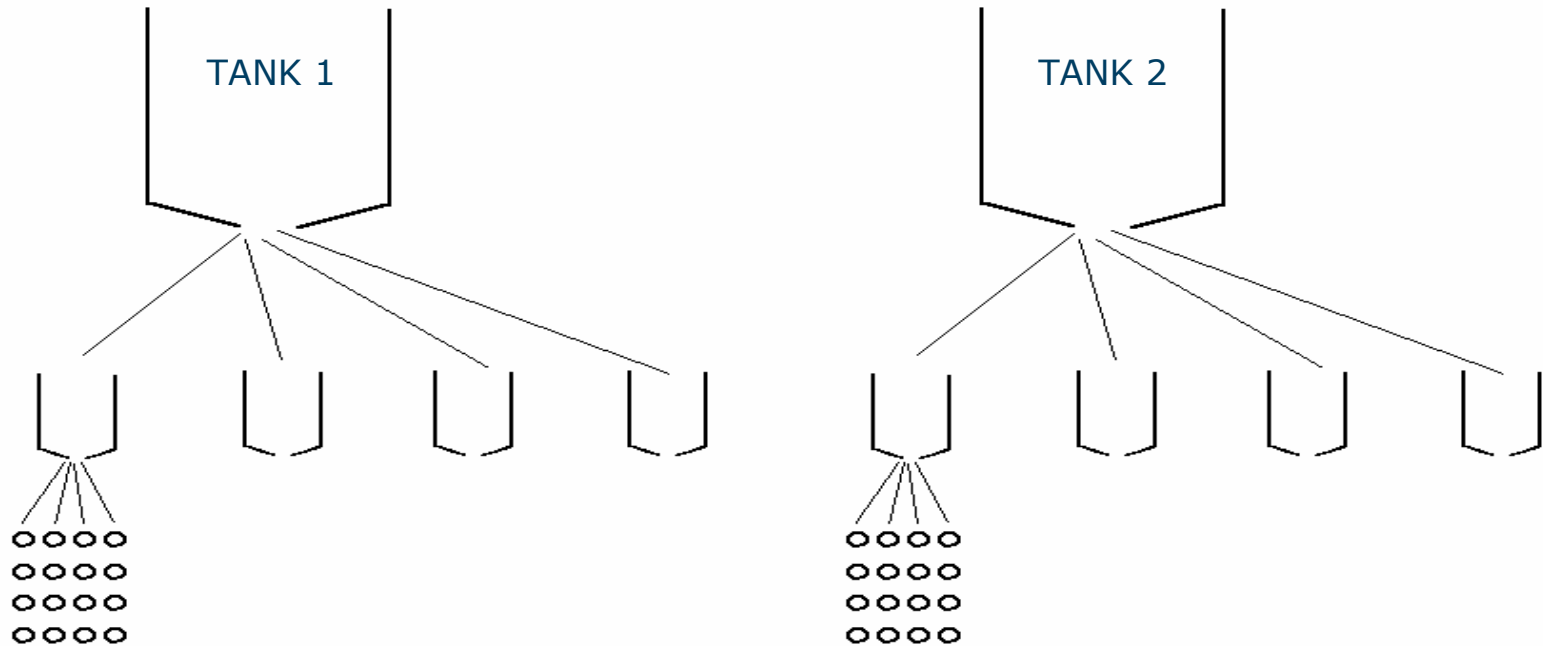
storage tanks (2 factors)

vats / curds (5 factors)

cheeses (3 factors)

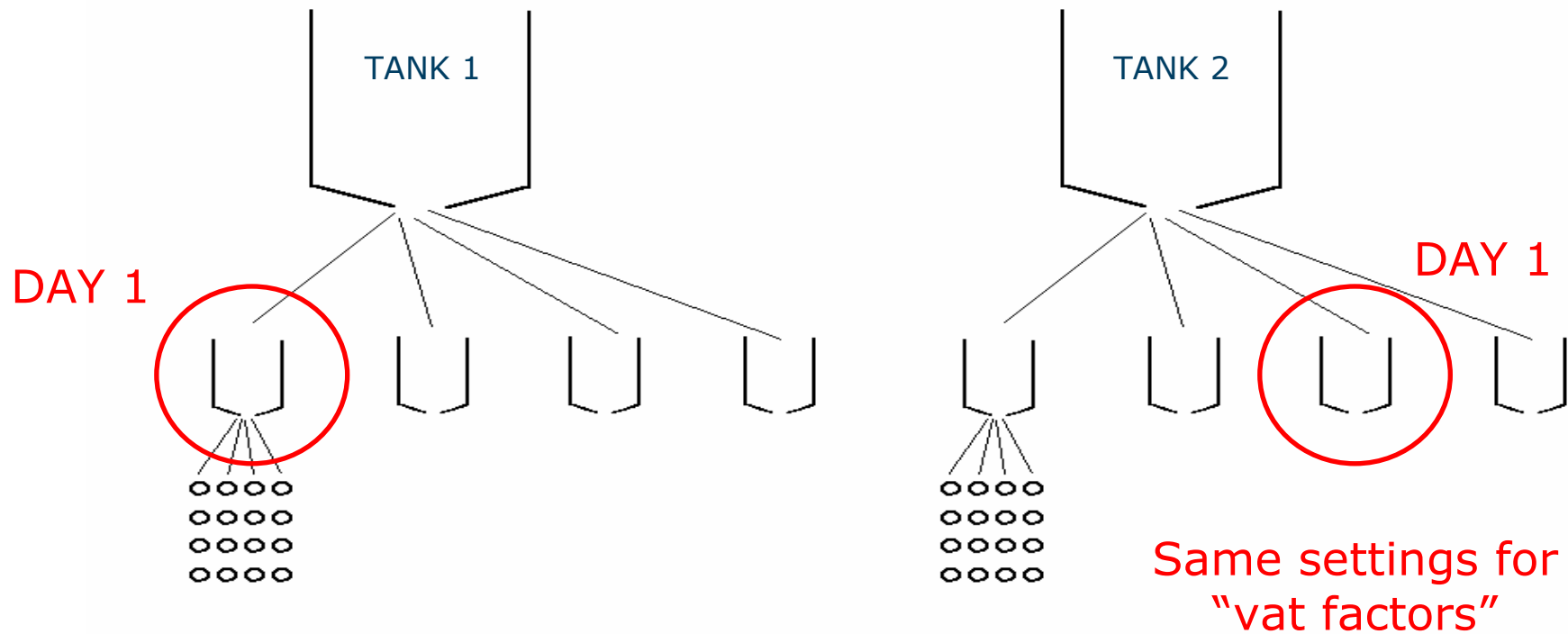


# Cheese-making experiment



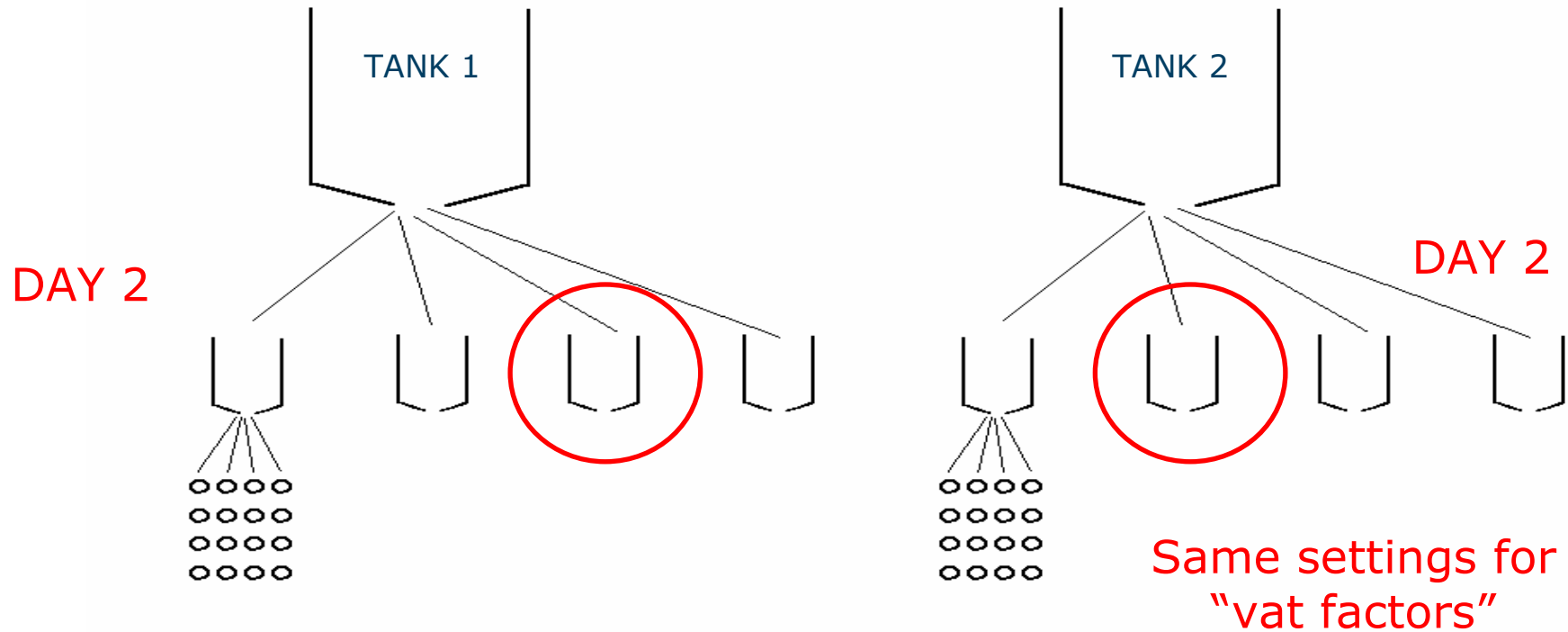


# Cheese-making experiment



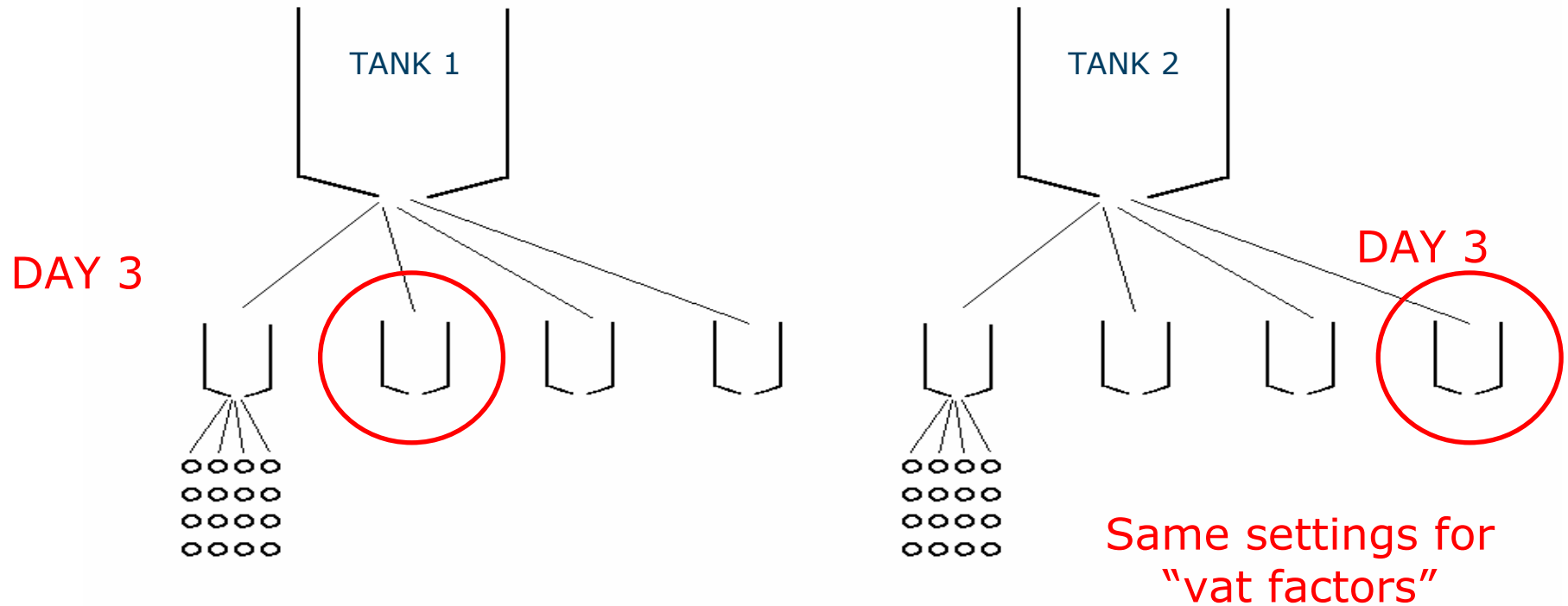


# Cheese-making experiment





# Cheese-making experiment





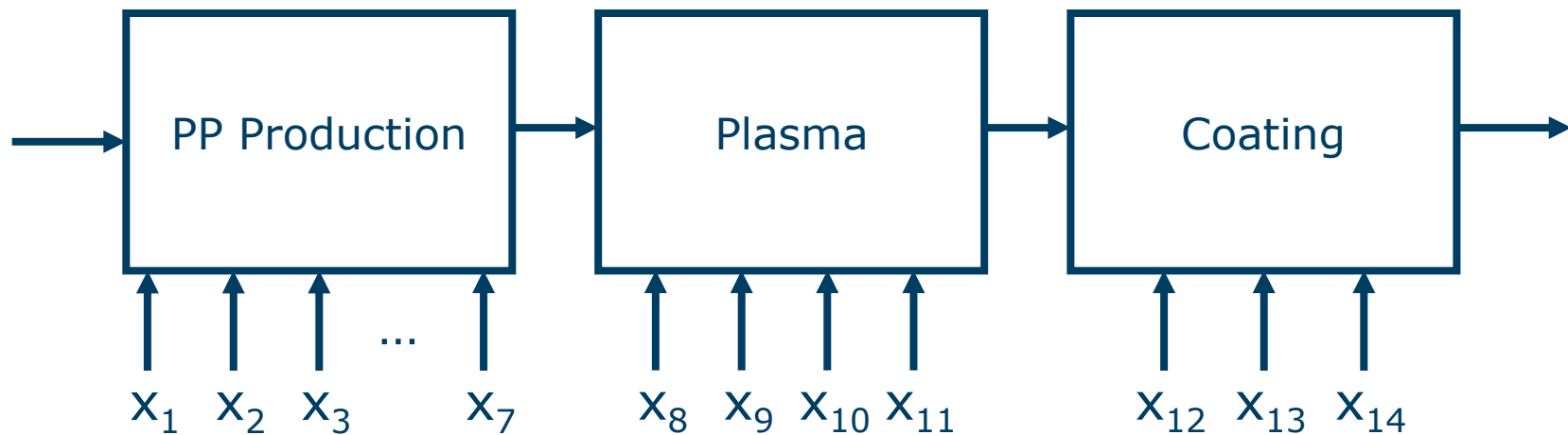
# Polypropylene experiment

- goal is to study adhesion of coatings to polypropylene
- production process involves 3 stages
  - stage 1: production of polypropylene  
7 factors
  - stage 2: gas plasma treatment  
4 factors
  - stage 3: coating  
3 factors



# Polypropylene experiment

- three process stages
- 14 factors





# Polypropylene experiment Stage 1

- 20 different polypropylene batches
- every batch contains a lot of plates
- 7 factors
  - homopolymer/copolymer
  - EPDM
  - talcum
  - mica
  - lubricant
  - UV-stabiliser
  - EVA
- all factors studied at 2 levels



# Polypropylene experiment Stage 2

- gas plasma treatment is done in an oven
- the oven can accommodate 5 different types of plates (4 repeats are used per type)
- 4 factors
  - type of gas
    - 2 activation gases
    - 1 etching gas
  - gas flow
  - power
  - reaction time
- all factors studied at 3 levels



# Polypropylene experiment

## Stages 1 and 2

Factors	Range/Levels	Unit
EPDM	0-15	%
Ethylene	0-10	%
Talcum	0-20	%
Mica	0-20	%
Lubricant	0-1.5	%
UV-stabiliser	0-0.8	%
EVA	0-1.5	%
Flow rate	1000-2000	sccm
Power	500-2000	Watt
Reaction time	2-15	min
Gas type	Etching gas Activation gas 1 Activation gas 2	

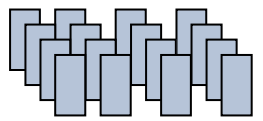
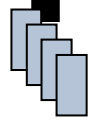
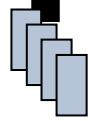
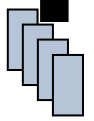
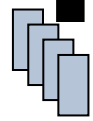
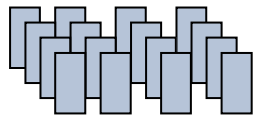
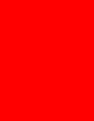
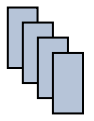
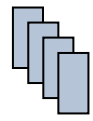
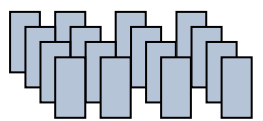
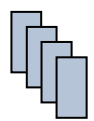
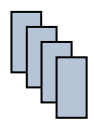
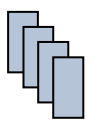


# Polypropylene experiment Stage 3

- coatings are applied to individual plates
- 3 factors
  - type of coating
    - solvent-based
    - water-based
    - UV-dried
  - quality (high end or low end product)
  - number of layers (one or two)
- in total, there are 12 types of coating

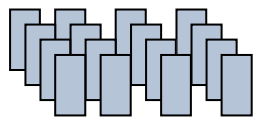
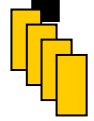
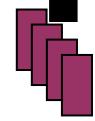
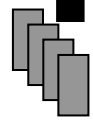
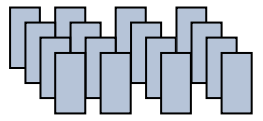
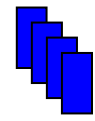
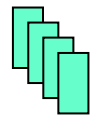
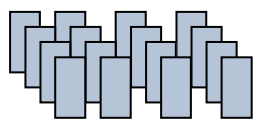

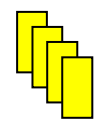


# Polypropylene experiment

Run	Polyprop. Batches	Gas Plasma Treatments			
		(1)	(2)	...	(24)
(1)	 → 				
(2)	 → 				
⋮	⋮	⋮	⋮	⋮	⋮
(20)	 → 				



# Polypropylene experiment

Run	Polyprop. Batches	Gas Plasma Treatments			
		(1)	(2)	...	(24)
(1)	 →	 ↑	 ↑		 ↑
(2)	 →				
⋮	⋮	⋮	⋮	⋮	⋮
(20)	 →				



# Mixture-process variable experiment

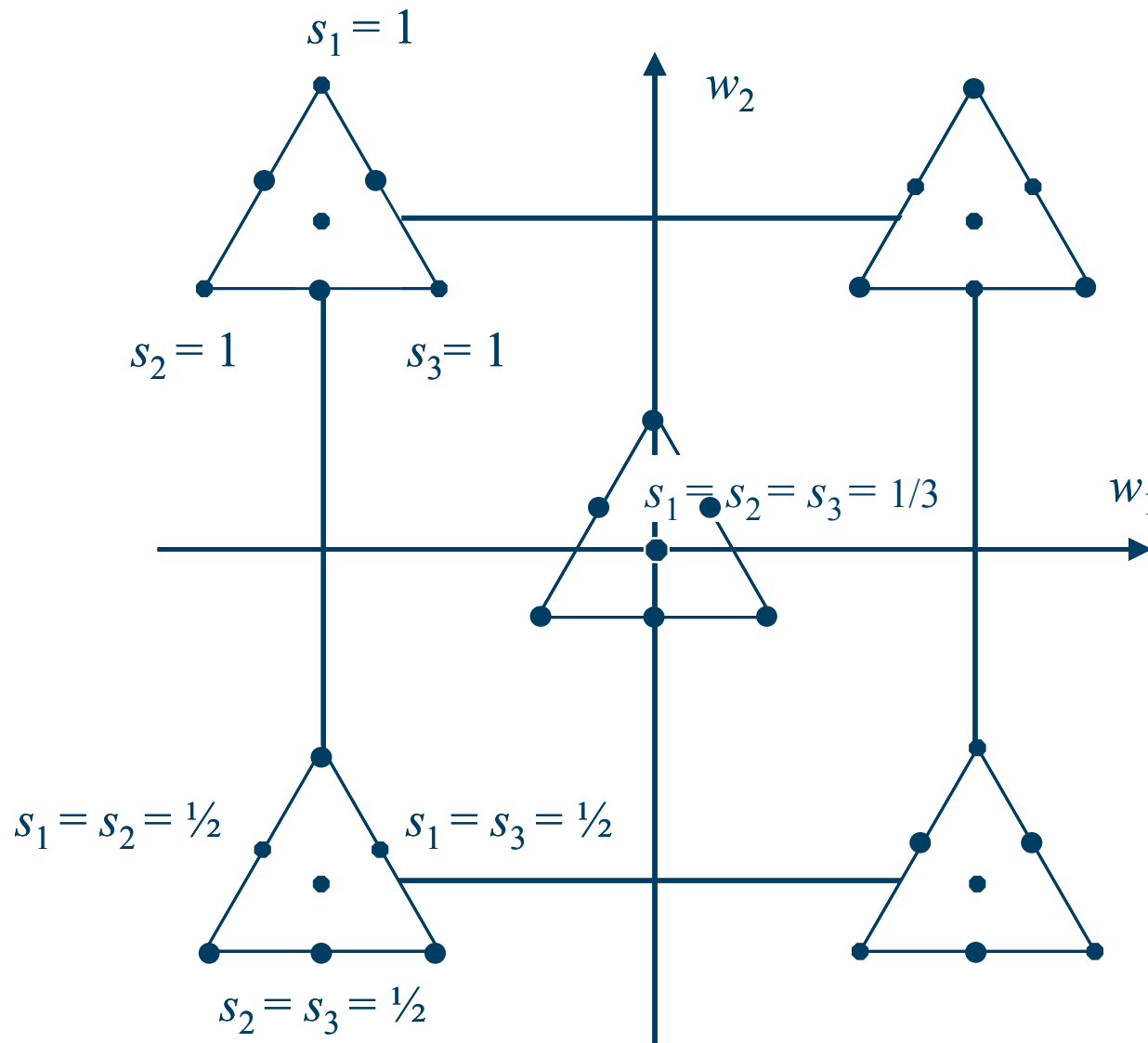
- vinyl thickness experiment (Kowalski, Cornell & Vining, Technometrics 2002)
- automobile seat covers
- 2 process variables:
  - rate of extrusion ( $w_1$ )
  - temperature of drying ( $w_2$ )
- 3 mixture variables
  - 3 plasticizers  $s_1$ ,  $s_2$  and  $s_3$
  - no constraint on proportions



# Mixture process-variable

# design

(35 runs)





# Mixture process-variable design

(35 runs)

Mixtures			Process Variables				
			-	+	-	+	0
S1	S2	S3	-	+	-	+	0
100%	0%	0%	X	X	X	X	X
0%	100%	0%	X	X	X	X	X
0%	0%	100%	X	X	X	X	X
50%	50%	0%	X	X	X	X	X
50%	0%	50%	X	X	X	X	X
0%	50%	50%	X	X	X	X	X
33%	33%	33%	X	X	X	X	X

W1  
W2





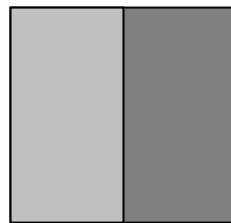
# Agricultural origin



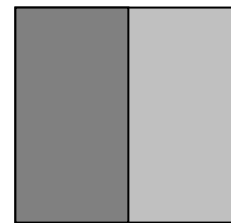
Field 1



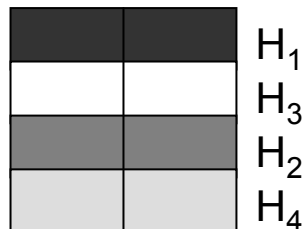
Field 2



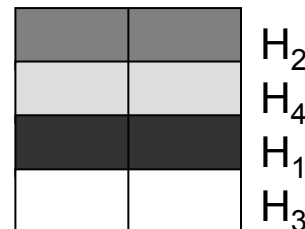
F<sub>1</sub> F<sub>2</sub>



F<sub>2</sub> F<sub>1</sub>





















F<sub>1</sub> F<sub>2</sub>



F<sub>2</sub> F<sub>1</sub>

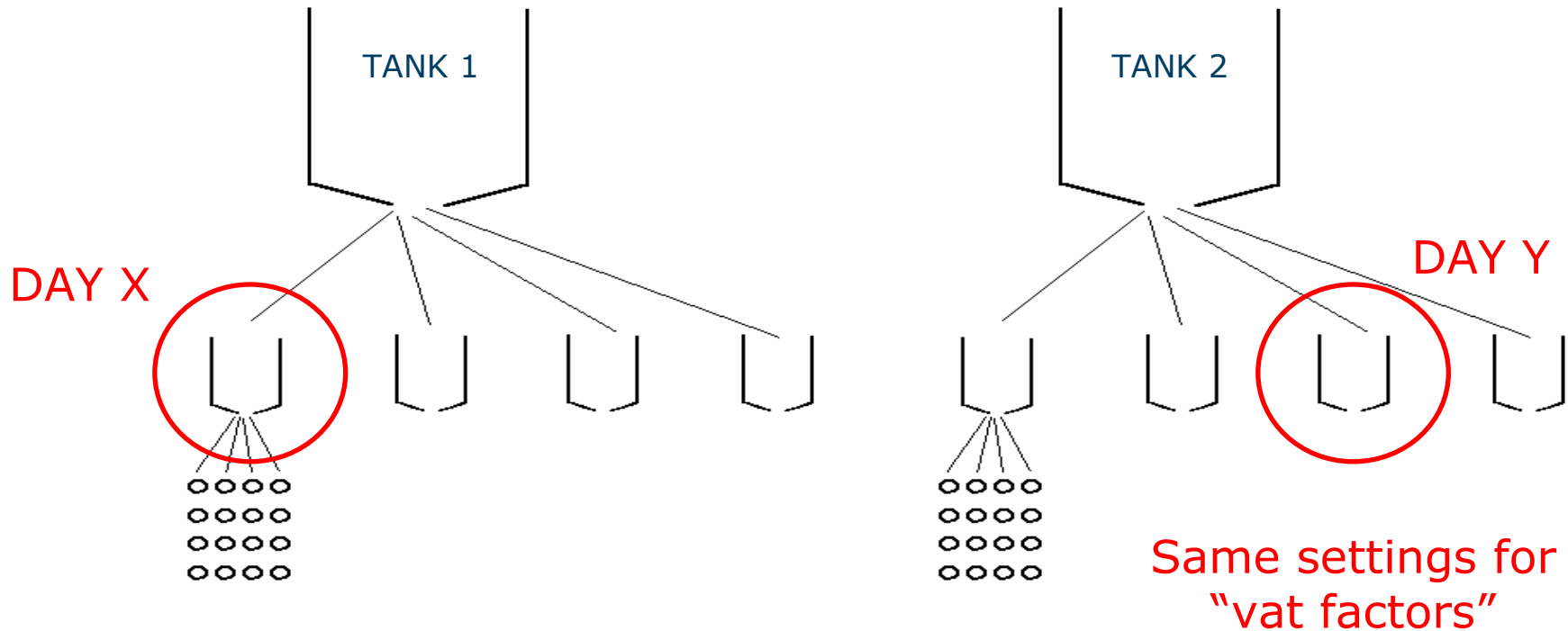


# The battery-cell experiment using a split-plot design?

Run	Assembly Variables ( $2^4$ )	Storage Variables ( $2^2$ ) Storage Conditions			
		(1)	(2)	(3)	(4)
(1)	 → 				
(2)	 → 				
⋮	⋮	⋮	⋮	⋮	⋮
(16)	 → 				



# The cheese-making experiment using a split-split-plot design?





## Battery-cell experiment

- assumption of independence is not valid
- 4 “row” factors, 2 “column” factors
- one observation per cell

$$Y_{ij} = \beta_0 + \sum_{k=1}^4 \beta_k^r X_{ik}^r + \sum_{k=1}^2 \beta_k^c X_{jk}^c + \sum_{k=1}^3 \sum_{l=k+1}^4 \beta_{kl}^r X_{ik}^r X_{il}^r \\ + \beta_{12}^c X_{j1}^c X_{j2}^c + \sum_{k=1}^4 \sum_{l=1}^2 \beta_{kl}^{rc} X_{ik}^r X_{jl}^c + \gamma_i + \delta_j + \varepsilon_{ij}$$

- $\gamma_i$ : random effect of the  $i$ th row
- $\delta_j$ : random effect of the  $j$ th column
- $\varepsilon_{ij}$ : random error



Model

## Polypropylene experiment

- 7 “row” factors, 4 “column” factors, 3 “cell” factors
- 5 observations per cell

$$\begin{aligned} Y_{ijk} = & \beta_0 + \sum_{l=1}^7 \beta_l^r X_{il}^r + \sum_{l=1}^4 \beta_l^c X_{jl}^c + \sum_{l=1}^6 \sum_{m=l+1}^7 \beta_{lm}^r X_{il}^r X_{im}^r \\ & + \sum_{l=1}^3 \sum_{m=l+1}^4 \beta_{lm}^c X_{il}^c X_{im}^c + \sum_{l=1}^7 \sum_{m=1}^4 \beta_{lm}^{rc} X_{il}^r X_{jm}^c + \sum_{l=1}^3 \beta_l^{cell} X_{jl}^{cell} \\ & + \sum_{l=1}^2 \sum_{m=l+1}^3 \beta_{lm}^{cell} X_{il}^{cell} X_{im}^{cell} + \sum_{l=1}^7 \sum_{m=1}^3 \beta_{lm}^{r,cell} X_{il}^r X_{im}^{cell} \\ & + \sum_{l=1}^4 \sum_{m=1}^3 \beta_{lm}^{c,cell} X_{il}^c X_{im}^{cell} + \gamma_i + \delta_j + \varepsilon_{ijk} \end{aligned}$$



## Model

- runs are not independent
- all the runs in the same row are correlated  
*because several observations were made without resetting the "row factors"*
- therefore, estimates of the effects of the row factors are imprecise
- all the runs in the same column are correlated  
*because several observations were made without resetting the "column factors"*
- therefore, estimates of the effects of the column factors are imprecise



## Model

- the interaction effects between the “row factors” and the “column factors” are estimated precisely
- therefore, strip-plot designs are ideal for robust product experiments where the primary interest is in interaction effects between control factors and noise factors  
(Box & Jones, J. Applied Statistics, 1992)
- effects of “cell factors” and all their interaction effects are estimated precisely too



## What if analyzed incorrectly

- main effects of the “row factors” and the “column factors” are declared statistically significant too often
- interaction effects between “row factors” are declared statistically significant too often
- interaction effects between “column factors” are declared statistically significant too often
- effects are also judged practically significant too often



# Model

$$Y = \mathbf{X}\beta + \mathbf{Z}_1\gamma_1 + \mathbf{Z}_2\gamma_2 + \varepsilon$$

$$E(\gamma_1) = \mathbf{0}_{b_1}$$

$$E(\gamma_2) = \mathbf{0}_{b_1b_2}$$

$$E(\varepsilon) = \mathbf{0}_n$$

and

$$\text{var}(\gamma_1) = \sigma_{\gamma_1}^2 \mathbf{I}_{b_1}$$

$$\text{var}(\gamma_2) = \sigma_{\gamma_2}^2 \mathbf{I}_{b_1b_2}$$

$$\text{var}(\varepsilon) = \sigma_{\varepsilon}^2 \mathbf{I}_n$$

$$\mathbf{V} = \text{var}(Y)$$

$$= \sigma_{\varepsilon}^2 \mathbf{I}_n + \sigma_{\gamma_1}^2 \mathbf{Z}_1 \mathbf{Z}_1^T + \sigma_{\gamma_2}^2 \mathbf{Z}_2 \mathbf{Z}_2^T$$

$$= \sigma_{\varepsilon}^2 \left( \mathbf{I}_n + \eta_1 \mathbf{Z}_1 \mathbf{Z}_1^T + \eta_2 \mathbf{Z}_2 \mathbf{Z}_2^T \right) \text{ where } \eta_i = \frac{\sigma_{\gamma_i}^2}{\sigma_{\varepsilon}^2}$$



# Model estimation

- generalized least squares regression

$$\hat{\beta} = \left( \mathbf{X}' \hat{\mathbf{V}}^{-1} \mathbf{X} \right)^{-1} \mathbf{X}' \hat{\mathbf{V}}^{-1} \mathbf{y}$$

- covariance matrix

$$\text{var}(\hat{\beta}) = \left( \mathbf{X}' \hat{\mathbf{V}}^{-1} \mathbf{X} \right)^{-1}$$

- $\hat{\mathbf{V}}$  is the estimated covariance structure of the observations
- $\hat{\mathbf{V}}$  can be estimated using restricted maximum likelihood estimation (REML)
- Kenward & Roger degrees of freedom



# Construction methods

- Combinatorial
  - Butler (Technometrics, 2004)
  - Miller (Technometrics, 2007)
  - Vivacqua & Bisgaard (Technometrics, 2009)
  - two-level designs
  - numbers of rows, columns and runs is power of two
- Computerized search
  - Arnouts, Goos & Jones (Quality and Reliability Engineering International, 2009)
  - D-optimal strip-plot designs
  - any types of factors, numbers of levels, ...
  - any numbers of rows, columns and runs



# Computerized search

- optimal experimental design
- D-optimality criterion: seeks designs that maximize determinant of information matrix

$$\mathbf{M} = \mathbf{X}' \mathbf{V}^{-1} \mathbf{X}$$

- information about the relative magnitude of variance components is required
- optimal designs are not sensitive to the specified values for the variance components



# Battery-cell experiment

(48 runs instead of 64)

Assembly factors				Storage factors				E F
				-	+	-	+	
A	B	C	D	-	-	+	+	
-	-	-	-	X	X	X		
+	-	-	-	X	X		X	
-	+	-	-	X		X	X	
+	+	-	-		X	X	X	
-	-	+	-	X	X	X		
+	-	+	-	X	X		X	
-	+	+	-	X		X	X	
+	+	+	-		X	X	X	
-	-	-	+	X	X	X		
+	-	-	+	X	X		X	
-	+	-	+	X		X	X	
+	+	-	+		X	X	X	
-	-	+	+	X	X	X		
+	-	+	+	X	X		X	
-	+	+	+	X		X	X	
+	+	+	+		X	X	X	



# Battery-cell experiment

(48 runs instead of 64)

Assembly factors				Storage factors						E
				-	+	-	+	+	+	
A	B	C	D	-	-	+	+	-	+	
-	-	-	-	X		X	X			
+	-	-	-		X			X	X	
-	+	-	-		X			X	X	
+	+	-	-	X		X	X			
-	-	+	-		X			X	X	
+	-	+	-	X		X	X			
-	+	+	-	X		X	X			
+	+	+	-		X			X	X	
-	-	-	+		X			X	X	
+	-	-	+	X		X	X			
-	+	-	+	X		X	X			
+	+	-	+		X			X	X	
-	-	+	+	X		X	X			
+	-	+	+		X			X	X	
-	+	+	+		X			X	X	
+	+	+	+	X		X	X			



# Proof-of-concept example

(24 runs)

First process stage		Second process stage								
		$x_3$	$x_4$	$x_5$	$x_6$	$x_7$				
$x_1$	$x_2$	-1	1	1	-1	1	-1	-1	1	$x_3$
		-1	1	-1	1	1	-1	1	-1	$x_4$
		1	1	1	1	-1	-1	-1	-1	$x_5$
		-1	1	1	-1	-1	1	1	-1	$x_6$
		1	-1	1	-1	1	-1	1	-1	$x_7$
-1	1	X	X	X	X	X	X			
1	-1	X	X	X	X			X	X	
-1	-1	X	X			X	X	X	X	
1	1			X	X	X	X	X	X	



## References

- Arnouts, H., Goos, P. and Jones, B. (2010). Design and analysis of industrial strip-plot experiments, *Quality and reliability engineering international* **26**: 127-136
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- Vivacqua, C.A. and Bisgaard, S. (2004). Strip-block experiments for process improvement and robustness, *Quality engineering* **16**: 495-500
- Vivacqua, C.A. and Bisgaard, S. (2009). Post-fractionated strip-block designs. *Technometrics* **51**: 47-55